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INTERPLANETARY GAS. II. EXPANSION OF A MODEL SOLAR CORONA*

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ABSTRACT

The kinetic theory for the escape of planetary and stellar atmospheres, developed by Stoney, Jeans, and others, is generalized to include any height above the atmospheric critical level. The theory is applied to the solar corona and the interplanetary gas; this approach may be contrasted with the use of the hydrodynamic equation of motion (Parker) and the equations of heat conduction and hydrostatic equilibrium (Chapman). The hydrodynamic expansion of the corona must actually be limited by the rate of evaporation, and Parker's large expansion velocities, presumed to correspond to a "solar wind," result from an invalid assignment of an integration constant and an ambiguity inherent in the hydrodynamic solution. Also, the mean energy per particle, which defines an equivalent gas temperature, is considerably lower at the earth's orbit than in Chapman's conduction theory, which does not allow for the evaporative escape of the more energetic particles. The mean expansion velocity, electron density, and equivalent temperature computed with the kinetic theory for two extreme situations are plotted against distance from the sun, and limitations to this simple model are discussed

I. INTRODUCTION

Parker (1958, Sec. III) has discussed the expansion of an isothermal, spherically symmetric, non-rotating corona in which the inhibiting effect of a magnetic field on the radial motions of ions is neglected. Beyond several solar radii, the computed velocities of mass motion are of the order of 500–1500 km/sec, which were thought to correspond to the solar wind postulated principally on the basis of Biermann's (1951, 1952, 1957) interpretation of the motions of comet tails. Should the coronal temperature suddenly diminish at some radial distance r, the expansion velocity, w, would remain constant beyond that point, but otherwise the acceleration would continue indefinitely, giving an infinite velocity as $r \to \infty$. Parker (1958, Sec. IV) also uses this model to compute the mass and heat loss (by convection) from the corona and the energy balance within the corona.

In this paper Parker's theory is reviewed, and its predictions of the escape flux are compared with that for evaporation from the high-energy tail of the Maxwellian distribution. The evaporation theory is generalized to obtain the mean expansion velocity at any distance above the critical level. Also the density distribution and the mean kinetic energy per particle (which is used to define an equivalent temperature) are computed out to the earth's orbit.

II. HYDRODYNAMIC EXPANSION

An atmosphere in strict hydrostatic equilibrium is defined by the equation

$$\frac{d}{dr}(NkT) + \frac{NG\mathfrak{M}M}{r^2} = 0\,, \tag{1}$$

where N is the total particle density, M is the mean mass per particle (half the hydrogen mass for a proton-electron gas), $\mathfrak{M}$ is the mass of the star or planet, and r is the radial

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distance from the center of the body. If the density at some reference point r_0 is N_0, then an isothermal atmosphere obeying equation (1) will have a density, denoted by $N_h(r)$, of

$$N_h(r) = N_0 e^{-(\lambda_0 - \lambda)}, \tag{2}$$

where

$$\lambda(r) = \frac{G\mathfrak{M}M}{kT\,r} = \frac{v_{\rm esc}^2}{U^2}. \tag{3}$$

Here $\lambda_0 \equiv \lambda(r_0)$, $v_{\rm esc}(r)$ is the velocity of escape, and U $[=(2kT/M_{\rm H})^{1/2}$ for protons or H atoms] is the most probable velocity of particles in a Maxwellian distribution. In the case of an ionized gas it may be supposed, at first, that the escape velocity for protons would be the same as for H atoms and that M should therefore be $M_{\rm H}$ rather than $\frac{1}{2}M_{\rm H}$ in the second equality of (3). However, as long as the electron and ion temperatures are the same, the tendency of the electron gas to escape from the protons will set up a restraining electric field which is just sufficient to reduce the net attractive force on the protons to half the gravitational force. For electrons the escape velocity is increased by the electric field, so that $v_{\rm esc}/U$ is the same for protons and electrons (see van de Hulst 1953, pp. 306 ff.).

In using the concept of the velocity of escape, we omit any retarding effect produced by collisions. The effective cross-section in the corona is of the order of $Q = 10^{-17}$ cm^2 (van de Hulst 1953, p. 308), which leads to a density at the critical level of $N_e(r_0) = 1/Q\,H_0 \sim 10^6$ electrons/cm^3 (where H_0 is the local scale height); hence the solar exosphere may be considered to lie above $r = 2$ or 3 solar radii. In later discussion I shall adopt the critical level r_0 as the reference level. Pikelner (1950) has considered the possibility that the proton and electron temperatures may be different at large r. Such complications would affect the escape velocity, but in this discussion the Maxwellian temperature will be taken as constant.

Parker points out that the solar corona cannot be in strict hydrostatic equilibrium because the finite pressure at infinity predicted by equation (2) is much greater than the pressure expected for a cold interstellar gas. (He actually generalizes this statement to allow for an arbitrary temperature decrease with increasing r, as obtained, for example, from the heat-conduction equation by Chapman [1957]). Hence the atmosphere must continually expand according to the equation of motion,

$$NMw\frac{dw}{dr} = -\frac{d}{dr}(NkT) - \frac{NG\mathfrak{M}M}{r^2}, \tag{4}$$

and the equation of continuity, which yields

$$N(r)\,w(r)\,r^2 = N_0 w_0 r_0^2. \tag{5}$$

We now write $\psi = w^2/U^2$ and $\xi = r/r_0$, where r_0 may be any reference distance but will be taken at the critical level for comparison with kinetic theory. Then the equation of motion is

$$\frac{d\psi}{d\xi}\left(1 - \frac{1}{\psi}\right) = \frac{4}{\xi} - \frac{2\lambda_0}{\xi^2}. \tag{6}$$

The allowed velocity of expansion seems to depend uniquely on the temperature; setting $\psi = 1$ in equation (6) gives $\xi = \lambda_0/2$. This condition effectively provides an integration constant, so that equation (6) yields

$$\psi - \ln\psi = -3 - 4\ln\frac{\lambda_0}{2} + 4\ln\xi + \frac{2\lambda_0}{\xi}. \tag{7}$$

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For slow expansion velocities, $w^2 \ll U^2$, this equation may be written

$$w = U\left(\frac{\lambda}{2}\right)^2 e^{-\lambda+3/2} = w_0\left(\frac{\lambda}{\lambda_0}\right)^2 e^{(\lambda_0-\lambda)}\,, \tag{8}$$

where we substitute λ_0/λ for $\xi = r/r_0$ by equation (3). Combining equations (8) and (5), we obtain the hydrostatic-density law of equation (2). Thus, as one would intuitively expect, small expansion rates do not affect the density distribution, but, at great distances r, the velocity w will, by equation (7), become large, and the density will decrease more rapidly than in equation (2).

This hydrodynamic expansion might be presumed to be quite distinct from the evaporation of the high-velocity Maxwellian particles (Stoney 1898; Spitzer 1949; van de Hulst 1953). Indeed, the expansion appears to give an escape rate that could be much greater than for evaporation. The formula for the evaporation flux (particles/cm² sec) at the critical level may be written (see Sec. III).

$$\mathfrak{F}_{\text{ev}} = \frac{N_0}{2\sqrt{\pi}}\,U(\lambda_0+1)\,e^{-\lambda_0}\,. \tag{9}$$

Thus for $w^2 \ll U^2$ at the critical level (which implies $\lambda_0 \gg 1$), the ratio of hydrodynamic expansion to evaporation would appear to be[1]

$$\frac{\mathfrak{F}_{\text{ex}}}{\mathfrak{F}_{\text{ev}}} = \frac{e^{3/2}\sqrt{\pi}}{2}\,\frac{\lambda_0^2}{\lambda_0+1} = \frac{3.97\lambda_0^2}{\lambda_0+1}\,. \tag{10}$$

If true, the above considerations would have a profound effect on the escape of helium from the terrestrial atmosphere (Bates and McDowell 1957, 1960; Nicolet 1957; Shklovskii 1958*a*, *b*), as well as for ionized gas from the sun. We shall see, however, that the hydrodynamic treatment is actually another way of treating the evaporative escape and that in a real situation the expansion velocity does not increase monotonically with r.

Let us consider a few points about the hydrodynamic equations. First of all, the equations do not depend in any way on collisional processes and (other factors being the same) are as valid in the exosphere, where collisions are negligible, as in the lower atmosphere. Second, the integration constant was not arbitrary but depended uniquely on the temperature. Hence the expansion described seems to be merely a result of the thermal energy competing with, and overpowering, the gravitational potential at large distances r; at smaller distances an upward flow is established to satisfy the continuity condition. The hydrodynamic escape thus seems to differ in no essential way from evaporation.

It seems strange, therefore, that w should exceed U, but this result is a consequence of an incorrect assignment of the integration constant and of an ambiguity inherent in the solution. For a given ξ in equation (7), ψ has two possible values, one greater and one less than unity. Parker has taken the small ψ at $\xi < \lambda_0/2$, the large ψ at $\xi > \lambda_0/2$, and joined the two parts of the solution by forcing the integration constant[2] to give ψ $(\xi = \lambda_0/2) = 1$. To avoid unrealistic infinite velocities, one must instead select the small ψ for all values of ξ. This means that ψ passes through a maximum and never

[1] This expression assumes that the density, N_0, at the critical level is the same regardless of the escape rate Actually, upward diffusion below this level could be sluggish enough to limit the rate of escape of particles In the earth's thermosphere the finite rate of diffusion may play an important role in reducing the escape flux of helium. I am indebted to Professor D R Bates for these remarks

[2] In terms of eq (6), the condition $\xi = \lambda_0/2$ (which is equivalent to $\lambda = v_{\text{esc}}^2/U^2 = 2$) on the right side actually corresponds to $d\psi/d\xi = 0$ on the left and not to the condition $\psi = 1$ as assumed in the integration constant of eq (7)

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equals unity. In the next section we shall see that the expansion velocity for thermal evaporation (eq. [17]) does indeed have a maximum at $\lambda = 2$, provided that the equivalent temperature (as defined by the mean energy per particle) is not appreciably diminished by the escape of high-velocity particles. In the latter event the maximum occurs at slightly smaller λ. (Note, also, that, even with Parker's solution from eq. [7], the slope $d\psi/d\xi = 0$ at $\lambda = 2$.)

Now if the differential equation (6) is considered only for $\psi \ll 1$ and $U^2 \ll v_{\rm esc}^2$, it gives w equal to the right side of equation (8), where w_0 is no longer given by the central portion of equation (8) but can be uniquely determined from the evaporative theory. If we take $w_0 = \mathfrak{F}_{\rm ev}/N_0$ from equation (9), the expansion velocity becomes

$$w = \frac{U}{2\sqrt{\pi}} \left(\frac{\lambda}{\lambda_0}\right)^2 (\lambda_0 + 1)\; e^{-\lambda} \qquad (\lambda \gg 1)\,, \tag{11}$$

instead of the first equality in equation (8). This equation can be valid both above and below the critical level but breaks down in the exosphere[3] as $v_{\rm esc}$ approaches U. For that case we shall examine the evaporative theory in detail.

III. EVAPORATIVE ESCAPE

Equation (9) for the escape flux by evaporation was obtained simply by finding the net mean velocity in the outward direction for a Maxwellian distribution that is missing particles with $v > v_{\rm esc}$ incident from the upward hemisphere. That a Maxwellian distribution exists for the same temperature throughout the exosphere (where there are presumed to be no sources or sinks of heat) follows immediately from Liouville's theorem in statistical mechanics. Thus we have

$$w = \frac{2\pi}{U^3 \pi^{3/2}} \int_{v_{\rm esc}}^{\infty} \int_0^{\theta_0} e^{-v^2/U^2}\, v^3 \cos\theta \sin\theta\, d\theta\, d v\,, \tag{12}$$

where $\theta_0 = \pi/2$. Although equation (9) was quoted for $\lambda = \lambda_0$ (the critical level), it appears at first that it should be equally valid for greater heights, i.e., for $\lambda \leq \lambda_0$. However, with the density given by equation (2), it is clear that the flux $\mathfrak{F}_{\rm ev}$ decreases only in proportion to $\lambda + 1$, whereas it should decrease as λ^2 to satisfy the continuity.

Jeans (1925) pointed the way toward the explanation of this paradox.[4] Strictly speaking, an atmosphere in hydrostatic equilibrium would have an isotropic distribution of particles of all velocities at any height. But in calculating the escape flux we exclude those particles falling from infinity, which is justifiable by the fact that an infinite time would be required to establish hydrostatic equilibrium at infinite distance. Jeans pointed out that, by excluding particles only from the upward hemisphere, we do not count a hyperbolic orbit, such as that shown in Figure 1, as it crosses the shell of radius r at point A, but we do count it as it emerges at point B. Thus to exclude completely all particles from infinity, we must integrate the escape flux over only the solid angle of half-width θ_0, which is measured to the tangent of a hyperbola that just grazes the critical level. The value of θ_0 will be a function of $v(r)$, r, and r_0. The particles with $v > v_{\rm esc}$ that rise from the critical level will be observed at r to be entirely within the angle θ_0. In the same fashion the normalization factor of equation (12) must be modified

[3] In this paper we are using the simplification of a sharp critical level which defines the height at which collisions abruptly become unimportant. Pikelner (1950) has considered the possibility that particles below this level also have a finite chance of escaping, and he has modified the effective velocity of escape to allow for the retardation produced by collisions.

[4] An alternative refinement was offered by Milne (1923), who adopted a hydrostatic equilibrium and computed the escape flux with eq. (9), but with the density at large r modified from eq. (2) by the finite mass of the atmosphere within r.

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when an appreciable contribution to the hydrostatic density of equation (2) becomes composed of particles on trajectories from infinity.

The corrected expression for the mean velocity of expansion, then, is

$$w=\frac{U}{p(\lambda)\,2\sqrt{\pi}}\int_{\lambda}^{\infty} e^{-x}x\,[1-\mu_0^2(x)]\,dx, \tag{13}$$

where $x = v^2/U^2$, $\mu_0 = \cos\theta_0$, and the normalization factor is

$$p(\lambda)=\frac{2}{\sqrt{\pi}}\int_0^{\lambda} e^{-x}x^{1/2}dx+\frac{1}{\sqrt{\pi}}\int_{\lambda}^{\infty} e^{-x}x^{1/2}\,[1-\mu_0(x)]\,dx. \tag{14}$$

To evaluate these two equations, we require $\mu_0(x)$. The orbit of a particle that passes through B (Fig. 1) is uniquely determined by the perihelion distance r_0 and the eccen-

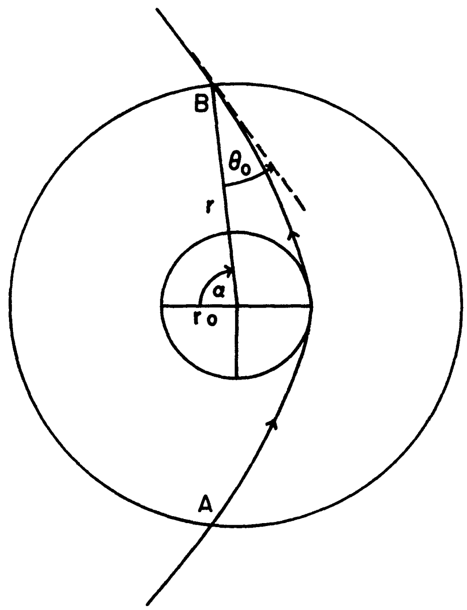

FIG. 1.—A hyperbolic orbit with velocity $v(r) > v_{esc}(r)$ enters sphere r at A and exits at B. The orbit is tangent to the critical level at r_0 and crosses the sphere r at an angle $\theta_0(v, r, r_0)$.

tricity, which is related to the total (kinetic plus potential) energy. The equation of a hyperbolic orbit, then, is

$$\frac{r}{r_0}=\frac{2x_0/\lambda_0}{1-\{1+[4(x-\lambda)\,x_0/\lambda_0^2]\}^{1/2}\cos\alpha}, \tag{15}$$

where the total energy, $x-\lambda = x_0-\lambda_0$, is a constant and where α is measured as indicated in Figure 1. Using this equation to derive the slope of the orbit at distance r, we obtain

$$\mu_0^2(x)=1-\frac{\lambda^2}{\lambda_0^2}\left[1+\frac{(\lambda_0-\lambda)}{x}\right] \qquad (x\geq\lambda). \tag{16}$$

Substituting equation (16) in equation (13) gives

$$w=\frac{U}{p(\lambda)\,2\sqrt{\pi}}\left(\frac{\lambda}{\lambda_0}\right)^2(\lambda_0+1)\,e^{-\lambda}. \tag{17}$$

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Since $p(\lambda) = 1$ (that is, the density is given by eq. [2]) when $U^2 \ll v_{\text{esc}}^2$, equation (11) is a special case of equation (17).

As λ becomes of the order of unity and smaller, the function $p(\lambda)$ decreases toward zero and greatly influences the behavior of w. In the asymptotic limit, as $r \to \infty$ and $\lambda \to 0$, $p(\lambda) \to 4\lambda^{3/2}/3\sqrt{\pi}$, and

$$w \to \tfrac{3}{8} U \frac{(1+\lambda_0)}{\lambda_0^2} \lambda^{1/2} . \tag{18}$$

The corrected density distribution becomes

$$N = N_h p(\lambda) , \tag{19}$$

where N_h is given by equation (2).

Another quantity of some interest is the mean-square velocity,

$$\langle v^2 \rangle = \frac{U^2}{p(\lambda)} \left\{ \frac{2}{\sqrt{\pi}} \int_0^\lambda e^{-x} x^{3/2} dx + \frac{1}{\sqrt{\pi}} \int_\lambda^\infty e^{-x} x^{3/2} [1 - \mu_0(x)] dx \right\} . \tag{20}$$

In the limit $r \to \infty$, we find

$$\langle v^2 \rangle \to U^2 \frac{9\sqrt{\pi}}{32} \frac{(1+\frac{2}{3}\lambda_0)}{\lambda_0^2} \lambda^{1/2} . \tag{21}$$

The mean-square velocity can be used to define an equivalent gas temperature. If T is the temperature of the corona at the critical level (that is, the parameter used in the Maxwellian distribution-curve at any point in the exosphere), the equivalent temperature is $T_{\text{eq}} = T \frac{2}{3}\langle v^2 \rangle / U^2$. Thus at large distances T_{eq} varies as $r^{-1/2}$; but it should be borne in mind that the velocity distribution is composed partly of motions directly outward from the sun.

IV. APPLICATION TO THE INTERPLANETARY GAS

Figure 2 shows an illustrative calculation for the correction factor for the density, $p(\lambda)$, the electron density, $N_e(=\frac{1}{2}N)$, the expansion velocity, w, and equivalent temperature, T_{eq}, as functions of λ and r/a, where a is the solar radius. The numerical values adopted for the critical level were $r_0 = 2.5a$, $T = 2 \times 10^6$ ° K, $N_e(r_0) = 10^6$ cm^{-3}, and $\lambda_0 = 1$. Since we have oversimplified the solar exosphere as beginning abruptly at a sharp critical level, there is a discontinuity in $p(\lambda)$ and the quantities depending on it, at r_0. This arises because λ is already of the order of unity when collisions become negligible.

At the earth's orbit ($r = 215a$), $\lambda = 0.012$, and about one-tenth the density is composed of gas escaping from the sun. The mean expansion velocity is about 20 km/sec, and the density is $N_e \approx 370$ cm^{-3}, in fair accord with values derived by Behr and Siedentopf (1953), Elsässer (1954), and Blackwell (1956*a*, *b*) from the zodiacal light.

It is of interest to compare these results with Chapman's (1957) model of the interplanetary gas, which was based on the equation of heat conduction. For a coronal temperature of 2×10^6 ° K, the temperature near the earth with Chapman's model would be about 440000° K. With the present exospheric model, we find $T_{\text{eq}} \approx 110000$° K—or about one-fourth Chapman's value—at the same distance. The difference must presumably be ascribed to a failure of the coefficient of heat conductivity to describe the heat flow when the velocity distribution becomes highly directional and non-Maxwellian. At great distances the discrepancy becomes worse, with Chapman's T falling as $r^{-2/7}$ and the exospheric T_{eq} varying as $r^{-1/2}$.

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Chapman computes the density with the equation of hydrostatic equilibrium for a radially decreasing T, whereas in this paper we have obtained the density by modifying the hydrostatic value for T = Constant to allow for the absence of particles falling from infinity. Chapman's density near the earth is quite sensitive to the temperature in the corona and, of course, depends on the coronal density at the point where heating stops. Although a reasonable choice of parameters gave a density near the earth in accord with the zodiacal-light results, the density gradient was quite different from the values indicated by those observations. In fact, his computed density passed through a minimum at $\frac{1}{2}$ or 1 a.u. and became infinite as $r \rightarrow \infty$ and $T \rightarrow 0$.

As a word of caution, it should be emphasized that the results in Figure 2 are for an

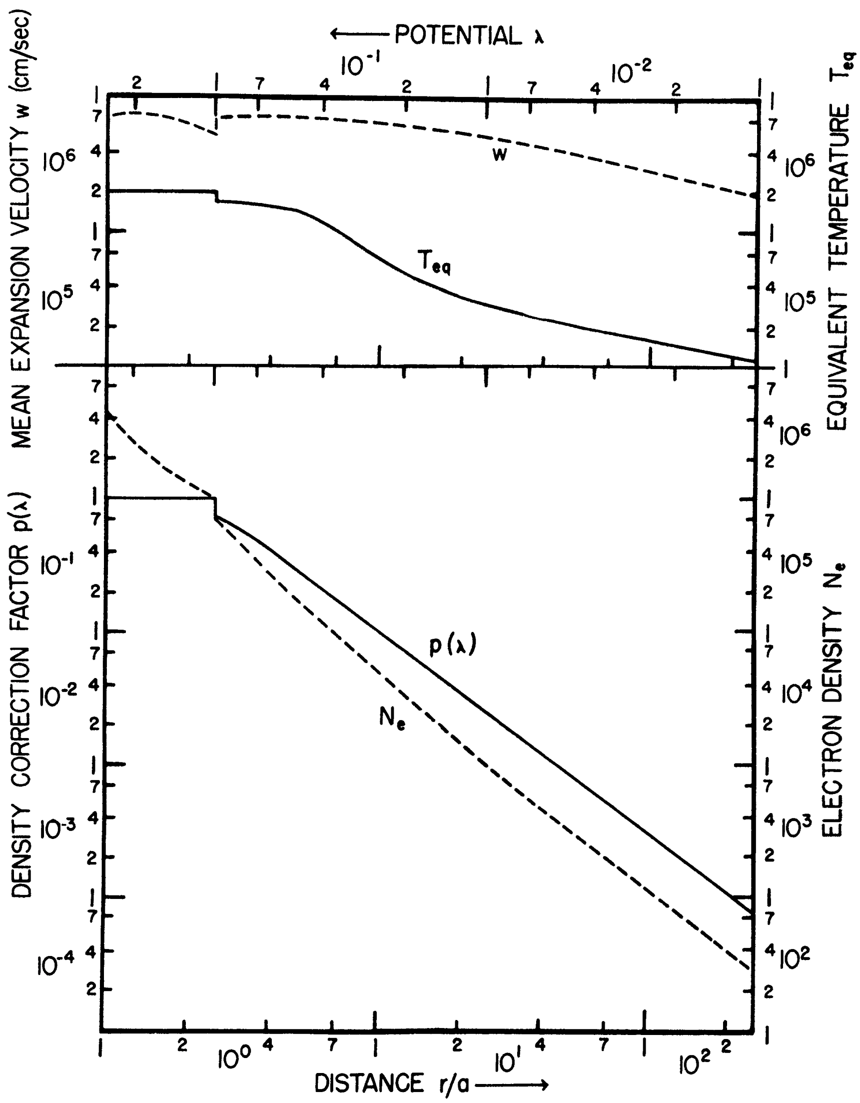

FIG. 2.—Illustrative values of the density correction factor, electron density, expansion velocity, and equivalent temperature versus distance from the sun and negative gravitational potential for the model corona. The discontinuities at the critical level and other departures from the real situation are discussed in the text The values are based on the assumption of a complete Maxwellian distribution below the critical level

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exospheric model and that departures of the real corona from this model may be important. For example, the velocity distributions employed here are based on the assumptions of no magnetic field and, especially, no collisions above the critical level and, of course, depend on the assumed coronal temperature. At some distance below the critical level the computed density becomes invalid because of the neglect of heavier ions, especially helium, and the assumption of a constant temperature. The true value of N_e must increase toward $r = a$ considerably faster than in a pure hydrogen corona. For the computed exospheric parameters to be reasonably correct, it is necessary that the solar magnetic field provide little or no restraint on the gas above the critical level, an assumption that seems not entirely unreasonable but certainly not proved.

Also we have assumed that a Maxwellian distribution is maintained immediately below the critical level, whereas the fast particles may actually become diminished through a rarity of close collisions and inadequate diffusion from below. This factor will serve to reduce the density, velocity, and equivalent temperature in the interplanetary gas. Of course, a finite expansion velocity below the critical level means that the velocity distribution is not strictly symmetric, but the outward flow of fast particles will tend to maintain the high-velocity tail at the critical level. The total loss of matter from the sun by evaporation would be $4\pi r^2 N_e(r)\, w(r) = 2 \times 10^{36}$ electrons/sec or 10^{20} gm/yr.

Pikelner (1950) has concluded, however, that the high-velocity particles will indeed become quite depleted and that the evaporative escape rate from the sun will be less than the limiting value computed above by a factor of 10^2 or 10^3. Therefore, we inquire what the density and equivalent temperature will be in the interplanetary gas in the limiting case of no net escape—that is, in the absence of any particles exceeding the escape velocity.

With only elliptical orbits in the outer corona, the normalization factor, now called $p'(\lambda)$, is given by only the first term in equation (14). Similarly, the mean-square velocity is obtained with only the first integral in equation (20). These values are readily computed with Pearson's (1922) tables of the incomplete Γ-functions. In Figure 3 are plotted $p'(\lambda)$, $N_e'(\lambda)$, and $T_{eq}'(\lambda)$, which apply for elliptical particle orbits only. At the earth's distance the electron density is still 90 per cent of the value including hyperbolic orbits. But the equivalent temperature, because it is more heavily weighted by the fast particles, is drastically reduced to the order of 10000° K, and at large distances varies as $1/r$ (i.e., $T_{eq}' \rightarrow \frac{2}{5}\lambda T$).

The time of flight of particles from the sun to the earth is sufficiently long that the velocity-distribution function may be altered through collisions and even become different for ions and electrons, with a consequent modification of the expansion velocity and density. The actual distribution is a matter still requiring investigation by Boltzmann's equation. Nevertheless, for treating the interplanetary gas, kinetic theory would seem to furnish a more appropriate first approximation than the equations of hydrodynamics or heat conduction.

In Paper I (Brandt and Chamberlain 1959) rocket measurements of Lyman-α in the night sky were interpreted in terms of a density in the interplanetary gas of at least 0.2 atom of neutral hydrogen/cm^3 in the earth's neighborhood. The density may be greater, depending on the profile of solar Lyman-α emission. This neutral hydrogen argues qualitatively in favor of a low temperature, of the sort given in Figure 3, and is definitely opposed to such a high temperature as 100000° K. The neutral density cannot, however, be derived on the basis of local (collisional or radiative) equilibrium but involves consideration of the time that is available for recombination before the particles fall back to the sun.

I am pleased to record that I had a number of provocative conversations on the topic of atmospheric escape with Professor D. R. Bates, whose interest proved most stimulating.

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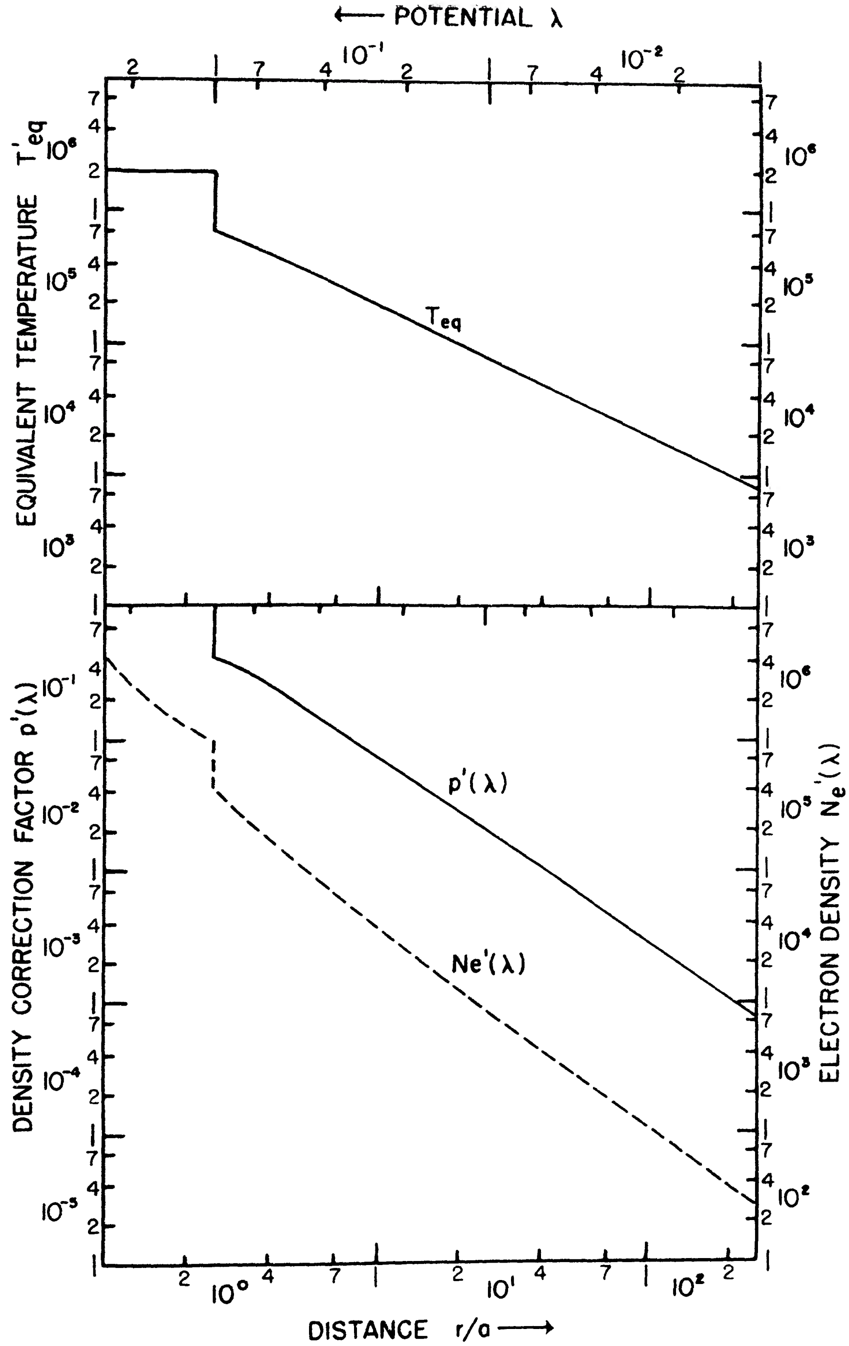

FIG 3.—Density correction factor, electron density, and equivalent temperature for the case of a Maxwellian distribution at the critical level for velocities below the escape velocity but with no particles exceeding the escape velocity.

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